Trajectory Generation for Stair Ascent Walking using Rayleigh Oscillator

T. Afzal, A.B. Wright and K. Iqbal

Department of Systems Engineering, 2801 S. University Ave, University of Arkansas at Little Rock, Little Rock, 72204, Arkansas.

Correspondence: txafzal@ualr.edu

Running Title: Trajectory Generation for Stair Ascent Walking using Rayleigh Oscillator

Abstract

This paper describes a trajectory generation technique for stair-ascent walking. The knee, hip and ankle joint trajectory during stair ascent are generated using mutually coupled, nonlinear oscillators. The parameters of the oscillators are tuned using the harmonic balance method, which converts the nonlinear differential equations to a set of algebraic equations. Fourier analysis of data generated by stair-ascent walking is performed to extract the amplitude and the phase of the dominant frequency components for each joint trajectory. The solution for the oscillator is assumed to be a sinusoidal wave and then by harmonic balance method the parameters of the oscillator are found. Each oscillator is responsible for generating a single frequency component with a specific phase and amplitude. The complete trajectory is obtained by summing the output of the oscillators that are relevant to one joint and the coupling maintains the phase relationship between the oscillators.

Introduction

Central Pattern Generators (CPG) consist of a group of neurons located in the spinal cord having the capability to generate sequences of cyclic excitation without feedback from the neuromusculoskeletal system and without the generation of control signals from the brain. The evidence of the existence of CPGs in humans and other vertebrates for cyclic motion generation such as walking or running has led to the notion of using neural oscillators for trajectory generation for the cyclic movements (Duysens et al. 1998).

Numerous research groups are investigating the behavior of CPGs in locomotion. The studies involve the application of CPG in bipedal, quadruple, hexapod and other n-paired leg animals. Bipedal locomotion contains numerous gait patterns such as walking, running and hopping.

Bay and Hemami (1987) used a Van der Pol (VDP) oscillator to generate various periodic wave patterns. Their thorough discussion on the properties of coupled oscillators with 3 nodes has shown that the oscillators can produce walking gait trajectories for the bipedal case, but they have not compared the CPG generated trajectories with the actual bipedal walking trajectories.

Zielinska (1996) investigated the application of a Van der Pol oscillator for bipedal level ground walking trajectory generation using oscillators with 4 nodes and provides a detailed account of the parameter changes that are required to change gait patterns. In addition, comparisons of the CPG generated trajectories with the natural bipedal walking trajectories were made. The results show differences in the CPG generated and natural gait trajectories. It is suggested that the addition of ankle joint angles should enable the generation of a more precise gait patterns.

Collins and Richmond (1994) have compared three different oscillator models: the Stein neuronal model, the VDP oscillator model, and the FitzHugh-Nagumo model. They demonstrated that a CPG model of coupled oscillators with 4 nodes can produce oscillation patterns corresponding to three common quadruped gaits – walk, trot and bound; however, their oscillators were only used for inter-limb control on a quadruped machine.

Liu et al. (2000) incorporate the ankle in the CPG network and use a fully connected ring network of VDP oscillators to generate trajectories of the hip, knee and ankle joints for one leg.

Dutra et al. (2003) and Pina Filho et al. (2005, 2009) propose a methodology to generate trajectories for level ground walking using the VDP, Rayleigh oscillator and a hybrid oscillator (combination of VDP and Rayleigh oscillator). They considered the simplest walking model that performs movement in the sagittal plane. The model has articulation at the hip joint and the knee joints. To solve the oscillator equations they have assumed the type of solution and determined the parameter values by substitution.

Nandi et al. (2009) used a Rayleigh oscillator to generate trajectories for the knee joint for level ground walking and applied it to an active knee prosthetic device. Their formulation was similar to Pina Filho et al.

(2005).

The ankle joint plays a vital role in executing movements that are performed against the action of gravity such as slope walking, jumping or even running. Reiner et al. (2002) conclude that the ankle, knee and hip joint all contribute positive power during stair ascent and descent (Fig. 1). It is important to generate trajectory for the ankle joint as it is applicable for bipedal robots and active ankle prosthesis for amputees.

Most CPGs to date have mainly focused on generating joint trajectories for level ground walking; however, traversing stairs or slopes are common activities that must be performed by active prostheses and walking robots. During stair climbing the trajectories of the hip, knee and ankle joint are different from level ground walking (Fig. 2).

Materials and Methods

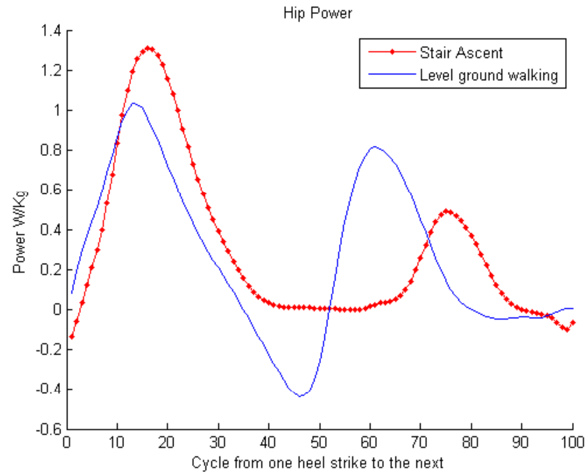
Bipedal gaits as well as the trajectories of the articulated joints during various gaits are periodic. As any periodic signal can be written as a sum of sine and cosine terms, by applying Fourier analysis to the trajectories of knee, hip and ankle joints the amplitude and relative phase of the dominant frequency component present in each trajectory can be determined. Data provided by Reiner (2002) was analyzed for the time-dependent trajectories of hip, knee and ankle joints for stair climbing (Fig. 2).

From the power spectrum of these data (Fig. 3) the dominant frequencies at 1 Hz, 2 Hz and 3Hz can be seen. For accurate extraction of frequencies a hanning window was used. The trajectories of the joints can be assumed to be a combination of these dominant frequencies. Thus each trajectory can be expressed as a sum of sinusoidal waves.

$$\theta_k = \theta_o + A_n \sum_{n=1}^{3} \cos(n\,wt + \propto_n) \qquad (1)$$

where θ_k represents the *kth* joint angle, A_n is the amplitude and $\propto_n$ is the phase of the *nth* frequency component.

Table 1 show the amplitude and phase relationship of the dominant frequency components for the ankle knee and hip joint.



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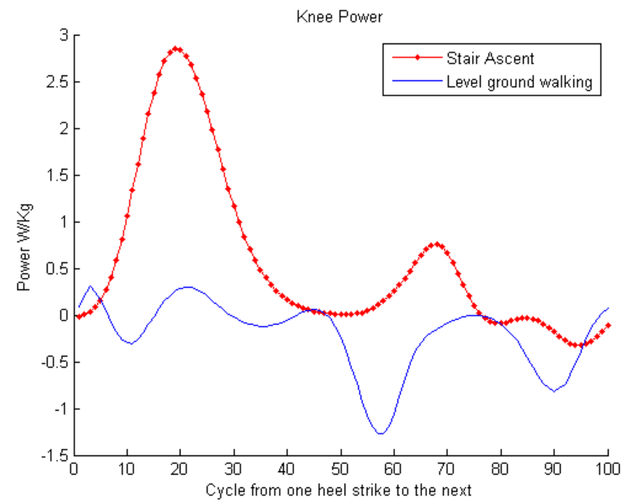

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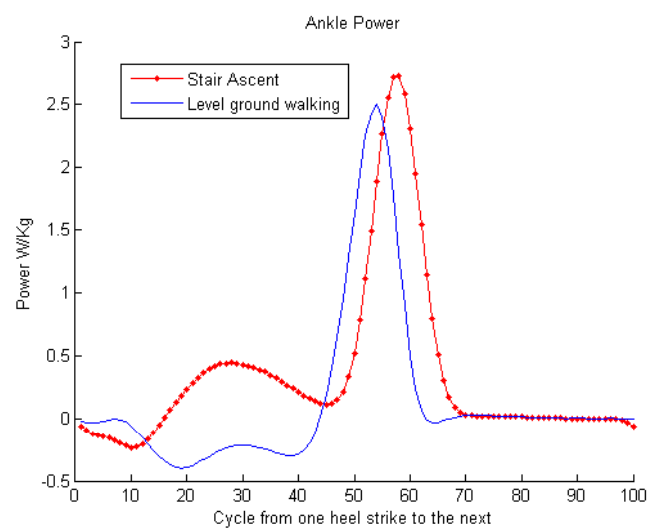
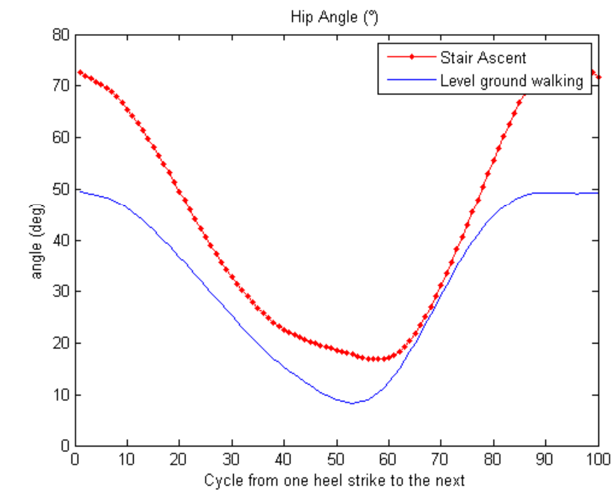


(c)

Figure 1. Joint powers during stair ascent and level ground walking. (a) Hip power. (b) Knee power. (c) Ankle power. (Data obtained from Reiner et al. 2002)



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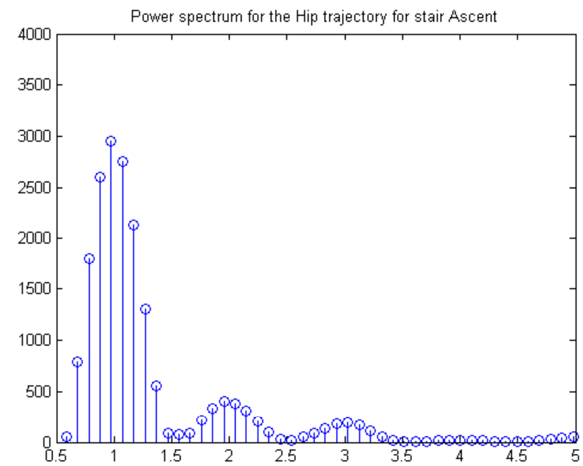

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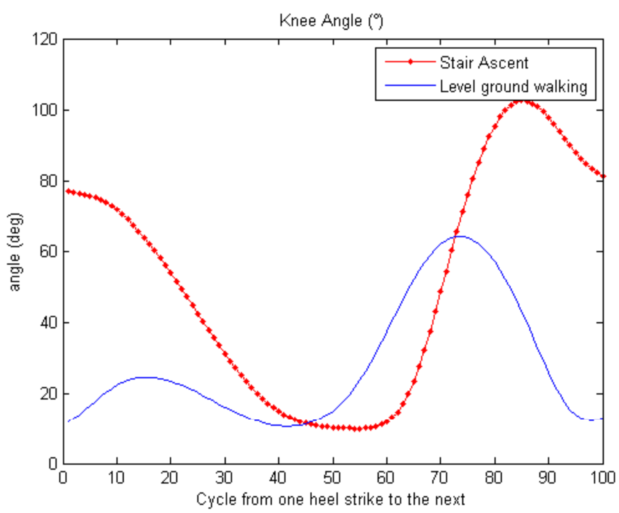

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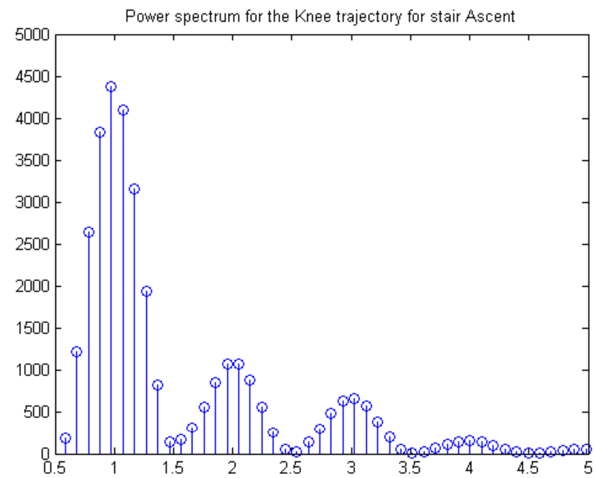

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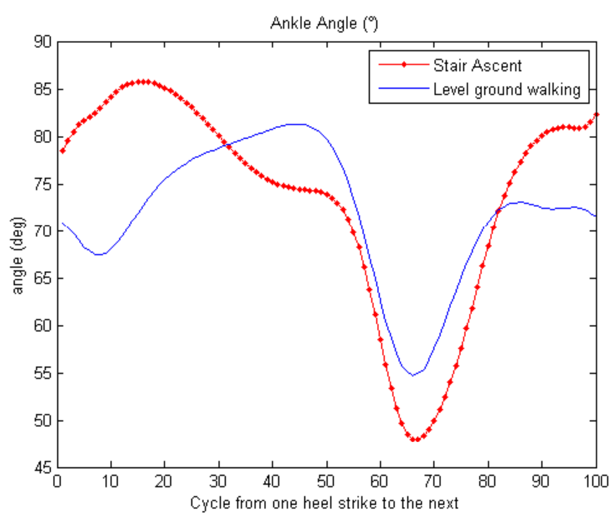

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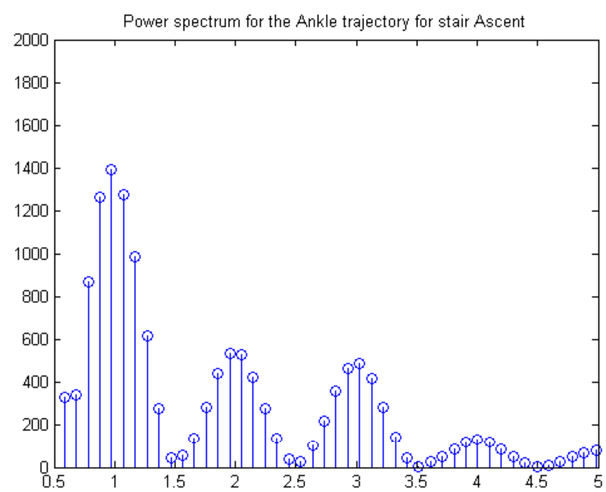

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Figure 2. (a) Hip, (b) knee and (c) ankle trajectories. Level ground walking and stair ascent. (Data obtained from Reiner et al. 2002).

Figure 3. Power spectrum of trajectories. Peaks are visible at 1 Hz, 2 Hz and 3 Hz (fundamental frequency 1 Hz).

Table 1: Trajectories relative phase and amplitude

Joint	Amplitude A	Phase (rad) $\propto$	Frequency (Hz) ω
Hip	30	0.1	1
	4	1	2
	-1.2	0.5	3
Knee	43	0.4292	1
	11	2.3292	2
	6	-2.4708	3
Ankle	14	-0.76	1
	5	-5.1	2
	4.5	-3.1	3

Rayleigh Oscillator

The oscillator model to be used for generating trajectories is the Rayleigh oscillator model introduced by British mathematical physicist Lord Rayleigh. The equation is of the form:

$$m\ddot{x} + kx = a\dot{x} - b(\dot{x})^3 \quad (2)$$

The equation of Rayleigh oscillator used in the analyses is of the form:

$$\ddot{\theta}_i - E_{in\omega}\left(1 - q_{in\omega}\dot{\theta}_i^2\right)\dot{\theta}_i + d_{in\omega}(\theta_i - \theta_{i0}) - \sum_{j=1}^{m} cjoint_{ij}\left[\dot{\theta}_j\left(\theta_j - \theta_{j0}\right)\right] - c_{irn\omega}\sum_{r=1}^{n1}\left(\dot{\theta}_i - \dot{\theta}_r\right) = 0 \quad (3)$$

where E_{iw}, q_{iw} and d_{iw} are the parameters of the Rayleigh equation and c_{ijw} and $cjoint_{ir}$ are the coupling coefficients of oscillator with the same frequency and oscillators with different frequency respectively.

Computing the first and second derivatives of equation (1), inserting the solution in equation (3) and applying the method of harmonic balance the values of the oscillator parameters $q_{in\omega}$ and $d_{in\omega}$ are obtained (equation (4)-(5)). By choosing appropriate values of the other parameters $E_{in\omega}, cjoint_{ij}$ and $c_{irn\omega}$ the trajectory can be computed.

$$q_{in\omega} = \frac{4}{3(n\omega)^2 A_{in\omega}^2} + \frac{1}{12(n\omega)^2 A_{in\omega}^2 d_{in\omega}} \sum_{i=1}^{m} A_{jm\omega}{}^2 cjoint_{ij} cos(\alpha_{in\omega} - 2\alpha_{jm\omega}) + \frac{1}{3(n\omega)^2 A_{in\omega}^2 d_{in\omega}} \sum_{r=1}^{n1} A_{rn\omega} - A_{rn\omega} c_{irn\omega} cos(\alpha_{in\omega} - \alpha_{rn\omega}) \quad (4)$$

$$d_{in\omega} = sqrt[(n\omega)^2[+\frac{m\omega}{2A_{in\omega}} \sum_{i=1}^{m} A_{jm\omega}{}^2 cjoint_{ij} cos(\alpha_{in\omega} - 2\alpha_{jm\omega}) - \frac{n\omega}{2} \sum_{r=1}^{n1} A_{rn\omega} - A_{rn\omega} c_{irn\omega} cos(\alpha_{in\omega} - \alpha_{rn\omega})] \quad (5)$$

Table 2: Parameters of the Rayleigh oscillator

Parameters		
E11 = 0.05	E21 = 0.3	E31 = 0.2
E12 = 0.05	E22 = 0.05	E32 = 0.5
E13 = 0.1	E23 = 0.2	E33 = 0.2
c121=0.005	c122=-0.06	c123=0.0
c131=0.005725	c132=0.015	c133=0.0
c211=-0.05	c212=0.8	c213=0.0
c231=-0.03	c232=-0.1	c233=0.0
c311=-0.03	c312=-0.12	c313=0.0
c321=0.018	c322=0.0052	c323=0.0
ch12=0.00	ck12=0.001	ca12=0.001
ch13=0.00	ck13=0.001	ca13=0.001
ch21=0.015	ck21=0.0001	ca21=0.0
ch23=0.015	ck23=0.0001	ca23=0.0
ch31=0.00	ck31=0.00	ca31=0.0
ch32=0.00	ck32=0.0	ca32=0.0

Coupling Scheme

The coupling between the oscillators is shown in Figure 4. As three dominant frequency components constitute the trajectory, each joint is composed of three mutually coupled oscillators; each oscillator is responsible for generating one frequency component with the relative phase and magnitude. The output trajectory is a sum of all the oscillator outputs relevant to the joint.

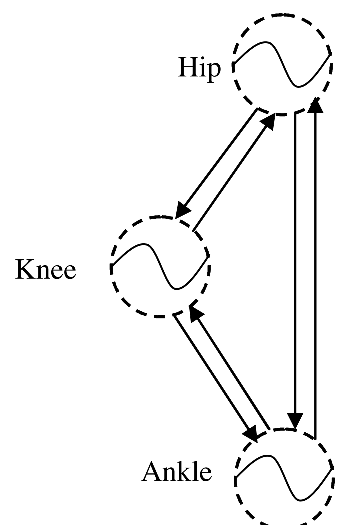
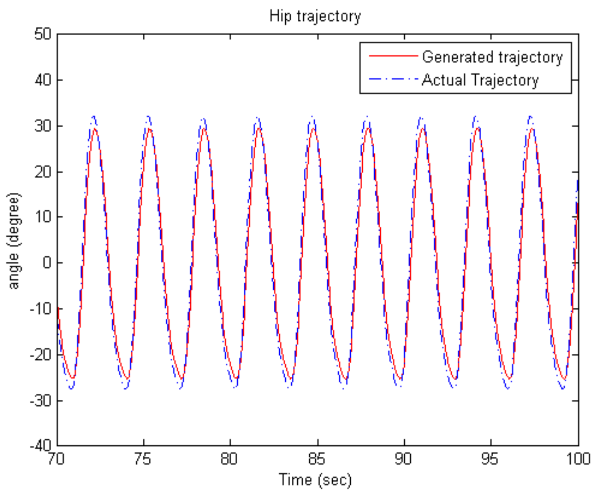


Figure 4. Oscillator coupling scheme for the bipedal case

Results

The gait patterns generated from the Rayleigh oscillators are shown in Figures 5, 6 and 7. The results were obtained by using the parameters in Table 1, calculating the oscillator parameters from equations 4 and 5 and then implementing equation 3 in Matlab. The values of the coupling parameters were obtained experimentally. The oscillator generated trajectories have been compared with the experimental data and we find approximate errors in the hip and ankle trajectories to be less than 5 degrees from 70-100 seconds of the trajectory plot. The knee trajectory also follows closely to the actual trajectory; however, a higher error between 15-20 degrees is found.

From the results it can be seen for the trajectories that over a period of 100 seconds the oscillator is successfully generating oscillations similar to reference trajectories.



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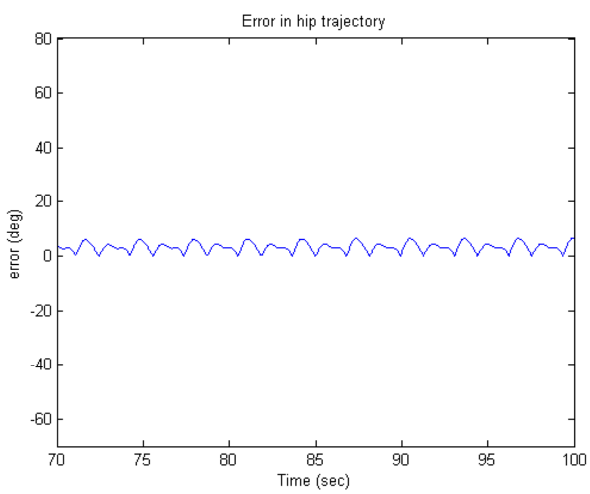
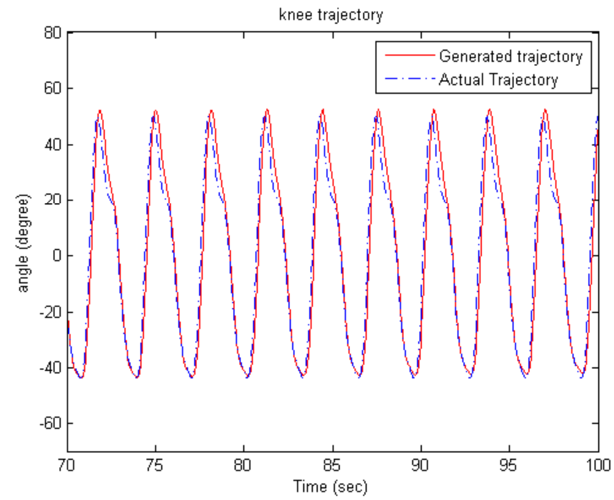


(b)

Figure 5. (a) Hip trajectory (b) Error.

Conclusion

Coupled non-linear Rayleigh oscillators were used to generate trajectories for the hip, knee and ankle joints for stair ascent walking. Fourier analysis of stair ascent data was done to extract the phase and amplitude of the dominant frequency components and the parameters of the Rayleigh oscillator were computed assuming a periodic solution and applying the method of harmonic balance. The results suggest that mutually coupled Rayleigh oscillators can be used to generate trajectories for the hip, knee and ankle joints for stair ascent walking.



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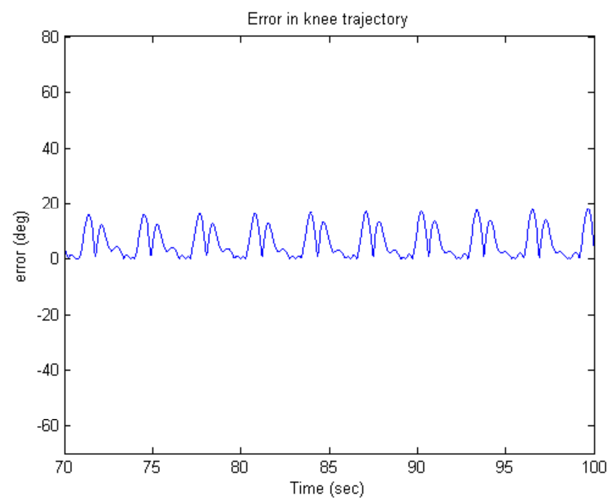


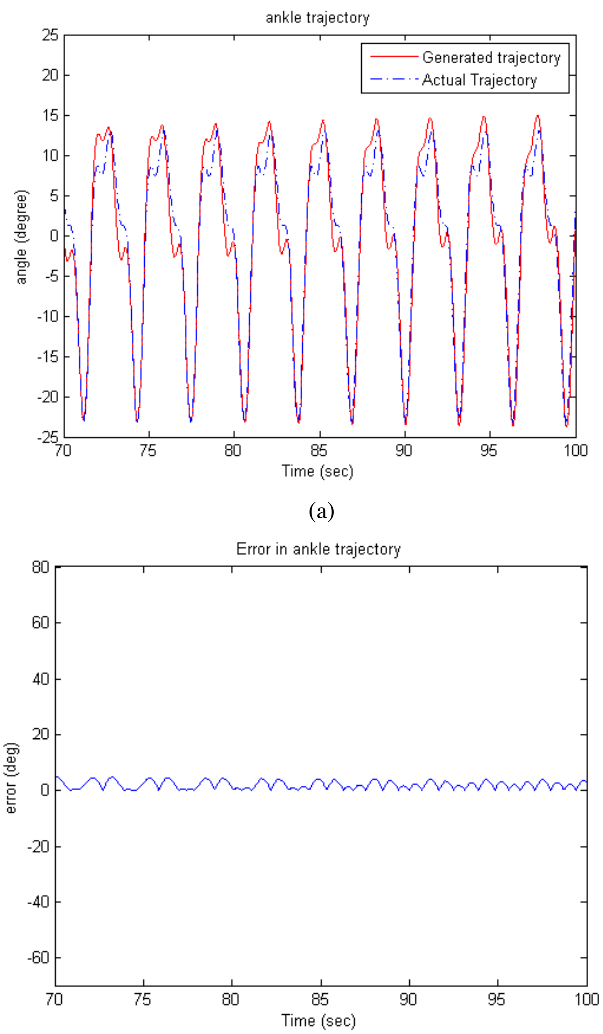
(b)

Figure 6. (a) Knee trajectory (b) Error.

Future Work

In this study the data analyzed was for a constant stair height with an inclination angle of 30 degrees. This is a standard angle of inclination implemented at public places. The varying stair inclination angles were not considered in this work. For trajectories from different inclination angles the Rayleigh oscillator is capable of generating those trajectories as the two parameters that vary for different trajectories are the amplitude and phase and the oscillator equations takes into account both of these parameters.

The next task is to investigate the transition between level ground and stair ascent walking. This is important as we do come across various walking paths in our daily activities especially frequent use of stairs and slopes at homes and at work places.



(a)

(b)

Figure 7. (a) Ankle trajectory (b) Error.

Once the transition is identified the Rayleigh oscillator would be tuned in such a manner that it has the capability to generate trajectories for both level ground walking Pina Filho et al. (2009) and stair ascent walking with instantaneous switching as required. This would require designing an adaptive Rayleigh oscillator. The adaptive mechanism would update the parameters of the oscillator to account for the transition from level ground to stair ascent walking.

Acknowledgements

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